DILATON STABILIZATION IN BRANE GAS COSMOLOGY[*]

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Brane Gas Cosmology is an M-theory motivated attempt to reconcile aspects of the standard cosmology based on Einstein's theory of general relativity. Dilaton gravity, when incorporating winding p-brane states, has verified the Brandenberger–Vafa mechanism —a string-motivated conjecture which explains why only three of the nine spatial dimensions predicted by string theory grow large. Further investigation of this mechanism has argued for a hierarchy of subspaces, and has shown the internal directions to be stable to initial perturbations. These results, however, are dependent on a rolling dilaton, or varying strength of Newton's gravitational constant G_N. In these proceedings we show that it is not possible to stabilize the dilaton and maintain the stability of the internal directions within the standard Brane Gas Cosmology setup.

1. Introduction

1.1. *Dilaton Gravity*

Dilaton Gravity comes from the low-energy effective action of Type II-A string theory, which is the result of M-theory compactified on S^1. Ignoring contributions from the bulk anti-symmetric two-form and including a potential $V(\phi)$ for the dilaton, the action of this system is described as

$$S = \int d^{10}x \sqrt{-G} e^{-2\phi} \left(R + 4G^{MN}\nabla_M \phi \nabla_N \phi - V(\phi) \right) , \qquad (1)$$

where G is the determinant of the background metric G_{MN}, ϕ is the dilaton, R is the Ricci-scalar.

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Assuming a spatially-homogeneous dilaton and an FRW-type metric of the form (2)[a], the action reduces to

$$ds^2 = -dt^2 + e^{2\lambda(t)} \sum_{i=1}^{3} (dx_i)^2 + e^{2\nu(t)} \sum_{j=1}^{3} (dy_j)^2 \tag{2}$$

$$S = \int d^{10}x \sqrt{-G_{00}} e^{-\varphi} G^{00} \left(-3\dot{\lambda}^2 - 6\dot{\nu}^2 + \dot{\varphi}^2 - V(\phi) \right) , \tag{3}$$

where the shifted dilaton $\varphi = 2\phi - 3\lambda - 6\nu$ is introduced for notational convenience.[2] Variation of the action with respect to $G_{00}, \phi, \lambda,$ and ν yields the system of equations

$$-3\dot{\lambda}^2 - 6\dot{\nu}^2 + \dot{\varphi}^2 = e^{\varphi} E + V(\phi) \tag{4}$$

$$\ddot{\varphi} - 3\dot{\lambda}^2 - 6\dot{\nu}^2 = \frac{1}{2} e^{\varphi} E + \frac{1}{4} V'$$

$$\ddot{\lambda} - \dot{\varphi}\dot{\lambda} = \frac{1}{2} e^{\varphi} P_{\lambda} - \frac{1}{4} V'$$

$$\ddot{\nu} - \dot{\varphi}\dot{\nu} = \frac{1}{2} e^{\varphi} P_{\nu} - \frac{1}{4} V' . \tag{5}$$

The scale-factor equations of motion in (5) indicate one of the substantial departures from general relativity: negative pressure terms will cause deceleration[b]. Indeed, it is precisely this last property that is exploited in Brane Gas Cosmology (BGC) to stabilize the internal dimensions; BGC provides the negative pressure source[6].

In the standard approach to BGC[3,5,6], energy E and pressure P_{λ}, P_{ν} contributions come from 1-branes, which are described by the Dirac-Born-Infeld (DBI) action, while $V(\phi) = 0$. The DBI action admits winding and momentum states, whose energy and pressure contributions are (respectively)

$$\begin{aligned} E_w &= \ \mu N e^{\lambda}, & P_w &= \ -\mu N e^{\lambda} \\ E_m &= \mu M e^{-\lambda}, & P_m &= \mu M e^{-\lambda}, \end{aligned} \tag{6}$$

where μ is the brane tension, N is the number of windings, and M is the number of momentum modes. The winding modes contribute the negative pressure P_w necessary to inhibit expansion of the internal directions.

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[a]The space is described by two subspaces with the intention of one subspace describing the large directions (λ), and the other describing the remaining internal dimensions (ν).
[b]Recall that in general relativity negative pressure terms are normally associated with inflation, not deceleration.

1.2. *The Rolling Dilaton*

We have remarked on one feature of dilaton-gravity which differs from that of general relativity; if, however, this theory is to be consistent with our current understanding of gravity, these differences must disappear and we must recover general relativity at some point in the early universe. The transition occurs in the case of a static dilaton: the action of dilaton-gravity, eq. (1), reduces to the Einstein-Hilbert action, where the vacuum value of the dilaton determines the strength of gravity as $G_N = e^{2\phi}/(16\pi)$. Thus, a rolling dilaton can be interpreted as a changing Newton's constant, which is strongly constrained on experimental grounds.[8]. If BGC is to make contact with our understanding of today's universe, the dilaton must stabilize at some point of its evolution — it is with this motivation that we have introduced a dilaton potential into the original action (1).

2. Dilaton Stabilization

2.1. *Perturbation Analysis*

To understand the effects of a stabilized dilaton we linearly perturb about a static solution by $\phi = \phi_0 + \delta\phi$, $\lambda = \lambda_0 + \delta\lambda$, and $\nu = \nu_0 + \delta\nu$. Such an expansion should draw similar conclusions to a perturbation about a slowly expanding universe since both the radion and dilaton must have masses much greater than the eventual Hubble rate. Due to our choice to expand about a static solution, and choosing $\phi_0 = \lambda_0 = \nu_0 = 0$, the zeroth-order equations of the system (5) imply the relations

$$E = -V(\phi)$$
$$E + 3P_\lambda + 6P_\nu = 8V'$$
$$P_\lambda = \frac{1}{2}V'$$
$$P_\nu = \frac{1}{2}V' \tag{7}$$

at the stationary point.

To understand the effect of small fluctuations about the stationary point, we expand the system (5) to first order. Keeping in mind that E, P_λ, and P_ν depend on λ and ν, we find that

$$\begin{pmatrix} \ddot{\phi} \\ \ddot{\lambda} \\ \ddot{\nu} \end{pmatrix} = \mathbf{S} \begin{pmatrix} \phi \\ \lambda \\ \nu \end{pmatrix} \tag{8}$$

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where the stability matrix $\mathbf{S}$ is given by

$$
\mathbf{S} = \begin{pmatrix}
2(V' - \frac{1}{2}V'') & -\frac{25}{8}V' + \frac{3}{4}\frac{\partial P_\lambda}{\partial \lambda} + \frac{3}{2}\frac{\partial P_\nu}{\partial \lambda} & -\frac{49}{8}V' + \frac{3}{4}\frac{\partial P_\lambda}{\partial \nu} + \frac{3}{2}\frac{\partial P_\nu}{\partial \nu} \\
\frac{1}{2}V' - \frac{1}{4}V'' & -\frac{3}{4}V' + \frac{1}{2}\frac{\partial P_\lambda}{\partial \lambda} & -\frac{3}{2}V' + \frac{1}{2}\frac{\partial P_\lambda}{\partial \nu} \\
\frac{1}{2}V' - \frac{1}{4}V'' & -\frac{3}{4}V' + \frac{1}{2}\frac{\partial P_\nu}{\partial \lambda} & -\frac{3}{2}V' + \frac{1}{2}\frac{\partial P_\nu}{\partial \nu}
\end{pmatrix}
\tag{9}
$$

This matrix is diagonalized by a similarity transformation, $\mathbf{P}^{-1}\mathbf{S}\mathbf{P}$, so that the general solution takes the form

$$
\begin{pmatrix} \phi \\ \lambda \\ \nu \end{pmatrix} = \mathbf{P} \begin{pmatrix} A_1 e^{i\omega_1 t} \\ A_2 e^{i\omega_2 t} \\ A_3 e^{i\omega_3 t} \end{pmatrix} + (\omega_i \to -\omega_i)
\tag{10}
$$

We insist that the eigenvalues ω_1 and ω_3 are real, giving a stable dilaton and radion, while ω_2 must be tuned to vanish (and the solution degenerates to $A_2 t + B$), so that the large dimensions are free to expand when the tuning (7) is relaxed.

However, the tuning of $\omega_2 = 0$ is not enough. The matrix elements $P_{\phi,2}$ and $P_{\nu,2}$ must be tuned to also vanish; otherwise ϕ and ν will mix with the unstable mode and stability of the dilaton and radion will be lost. Certain linear combinations of ϕ, ν and λ will oscillate, but this is not adequate: we really need ϕ and ν separately to settle to some fixed values. This can be accomplished by adjusting parameters so that $S_{\phi,\lambda} = S_{\nu,\lambda} = 0$. Unfortunately, this requires an exact cancellation between the first derivative of the dilaton potential and quantities ($dP_\nu/d\lambda$ and $dP_\lambda/d\lambda$) characterizing the brane gas, entities which have no reason to be related to each other. If the tuning fails by even a small amount, the contamination of the dilaton and radion by the mode which expands like the scale factor of the large dimensions will eventually lead to unacceptably large evolution of these fields.

Let us contrast this to the usual BGC scenario where the dilaton is rolling and has no potential. In the radion-stability analysis[6], the dangerous off-diagonal term $S_{\nu,\lambda} = \frac{1}{2}dP_\nu/d\lambda$ is taken to be zero, and there is no need to cancel it against V'. The pressure in the extra dimensions is assumed to be due to winding and momentum modes which exactly cancel each other at $\nu = 0$,

$$
P_\nu = -\mu N(e^\nu - e^{-\nu}).
\tag{11}
$$

Here μ is the tension of the brane, and therefore positive, and N is the number of winding modes. However $dP_\nu/d\nu$ is not zero, and has the right sign to stabilize the extra dimensions, since $S_{\nu,\nu} < 0$. If we try to use the

same approach but also stabilize the dilaton, then $V = V' = P_\lambda = P_\nu = 0$ at the stationary point, while V'' can be nonzero. We then have one vanishing eigenvalue, as needed for the growth of the large dimensions, while the other two eigenvalues are given by

$$-\omega_\pm^2 = \frac{1}{2}\left(\frac{1}{2}P' - V'' \pm \sqrt{\left(\frac{1}{2}P' - V''\right)^2 + \frac{1}{2}V''P'}\right) \qquad (12)$$

where $P' = dP_\nu/d\nu$. It can be shown that one of $\omega_\pm^2 < 0$ for any value of P' or V'', meaning that there is always one unstable mode in addition to the unstable $\omega = 0$ mode. This result does not contradict previous studies of the stability of the extra dimensions in BGC because these works were only concerned with the stability of the radion[6,7]. Instead, this result shows that stabilization of the dilaton and the radion is not possible without the introduction of some new potential for the radion, and it demonstrates the importance of a rolling dilaton to the stabilization mechanism in BGC.

3. Conclusions

The original proposal of Brandenberger and Vafa argues why, within a string theory context, only three dimensions will be able to grow large.[1] This idea has been extended to argue for a hierarchy of subspaces, has been numerically verified, and issues such the stability of the internal dimensions have been explored[3,5,6]. A remaining aspect of this scenario is to understand the evolution of the dilaton, an evolution which must stabilize at some later phase, thus marking the transition to general relativity. To this end, we have introduced a potential for the dilaton into the original action and tracked the evolution of perturbations about a static configuration.

The analysis indicates the importance of a rolling dilaton in order for the negative-pressure winding modes to inhibit growth of the extra dimensions. When the dilaton stabilizes, the original action reduces to the usual Einstein-Hilbert action of general relativity, where all sources act to accelerate expansion, causing the radion to grow. The eigenvalues of eq. (12) quantify this previous statement, indicating that any static solutions are unstable to perturbations, so that two unstable modes will always exist. Thus, solutions with one growing mode (corresponding to three directions growing large), and two stable modes (corresponding to a stable dilaton and radion) are physically implausible within the normal BGC setup.

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Although BGC can stabilize the radion during the dilaton-gravity epoch, some new mechanism must stabilize the internal dimensions once the dilaton has stopped rolling.[9] Presumably the onset of the dilaton potential will coincide with the appearance of a potential for the radion as well: one possibility is from compactifications with fluxes[4]. This possibility provides an interesting link between BGC and the work of Giddings, Kachru, and Polchinski and subsequent investigations.

Acknowledgments

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